LightMachinery
Excellence in Lasers and Optics
80, Colonnade Road
Ottawa, Ontario, K2E 7L2. Canada

Technical Note: Dependence of the Precision of Brillouin Shifts on the SNR of the Signal.

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For questions, contact hyperfine@lightmachinery.com

Introduction

A key specification of any Brillouin spectrometer is the ability of the spectrometer to accurately measure the *shift* (in frequency) induced by Brillouin scattering in the sample under investigation. The reproducibility of the measured shift is the *precision* of the measurement, as defined in [1]. The measurement technique described in [1] is paraphrased below:

"Each Brillouin spectrum was fitted with Lorentzian peaks using a least squares iterative method. The positions of the peaks were recorded, and a frequency shift calculated. This measurement was repeated multiple times, and the standard deviation of the multiple shifts was calculated. *The precision is equal to this standard deviation*."

This note relates the precision to the Signal-to-Noise Ratio, SNR, of the recorded Brillouin spectrum.

Theory

Equation (6) of [2] describes how the error in the measured Brillouin shift, $\delta\nu_B$, can be written as

$$(\delta\nu_B)^2 = \frac{\Gamma_B^2 + Ba^2}{N} + \frac{4\sqrt{\pi}\Gamma_B^3 b^2}{aN^2} \qquad \text{(A)}$$

where Γ_B is the natural Brillouin linewidth, N is the total number of Brillouin photons, B is a broadening factor, which describes pixelation noise, a is the spectral channel size, and b is the number of background noise photons. The first term contains the dependence of precision on the photon-counting noise coming from shot and pixelation noise, while the second term contains the background noise contribution. We will only consider situations where both pixelation noise and background noise are negligible compared to the first term in the above equation [3], [4]. The equation then simplifies to

$$\delta\nu_B = \Gamma_B/N^{0.5} \qquad \text{(B)}$$

Equation (B) above is very similar to the "Kitt Peak Criteria" which states that "*the criterion derived from the Kitt Peak work (shows that) wavenumber precision is of the order of FWHM divided by signal-to-noise ratio.*" [5]. In the case of signals that are dominated by shot-noise, the SNR is $N^{0.5}$, and Equation (B) and the Kitt Peak Criteria become identical.

$$\delta\nu_B = \Gamma_B/N^{0.5} = \Gamma_B/SNR \qquad \text{(C)}$$

where Γ_B is now the measured width (FWHM) of the Brillouin peak (the convolution of the natural Brillouin linewidth with the instrument function).

Experiment

Although Equation (C) was validated in [2] a series of experiments were carried out at LightMachinery to confirm that the relationship between precision and SNR holds for LightMachinery HF-9999 spectrometers. Brillouin shifts were measured in an acrylic sample and the precision determined at different exposure times from 10 msecs to 333 msecs. The peak Brillouin signal was found to be proportional to the exposure time of the spectrometer sensor, as shown in Fig. 1.

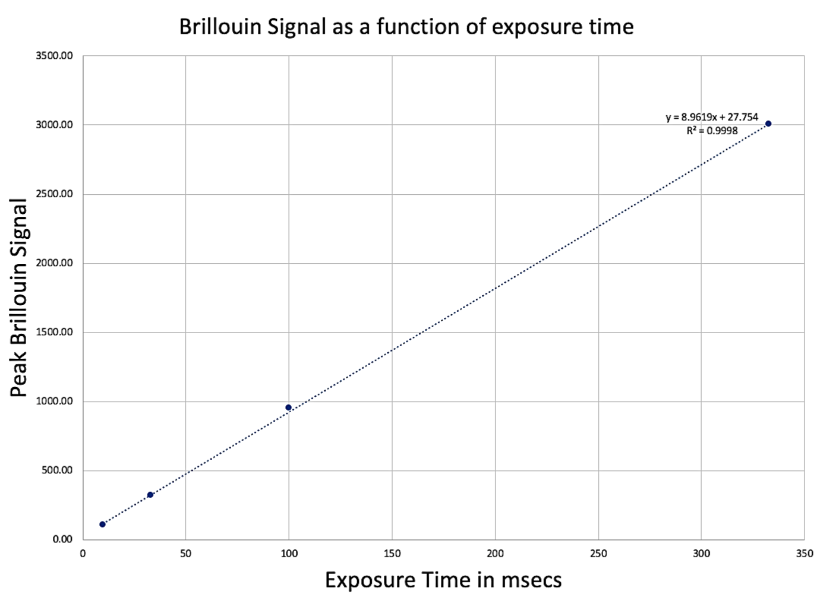

Figure 1. Plot of the peak amplitude of the Brillouin signal (arbitrary units) as a function of sensor exposure time (msec).

For all the measurements shown in Fig. 1, the photo-electron shot noise was much greater than either the pixelation noise, a, or the background noise, b. Hence, Equation (B) holds true, and the measured precision is expected to be proportional to 1 / (peak signal)$^{0.5}$, or 1 / (exposure time)$^{0.5}$. Figure 2 demonstrates that this is indeed the case.

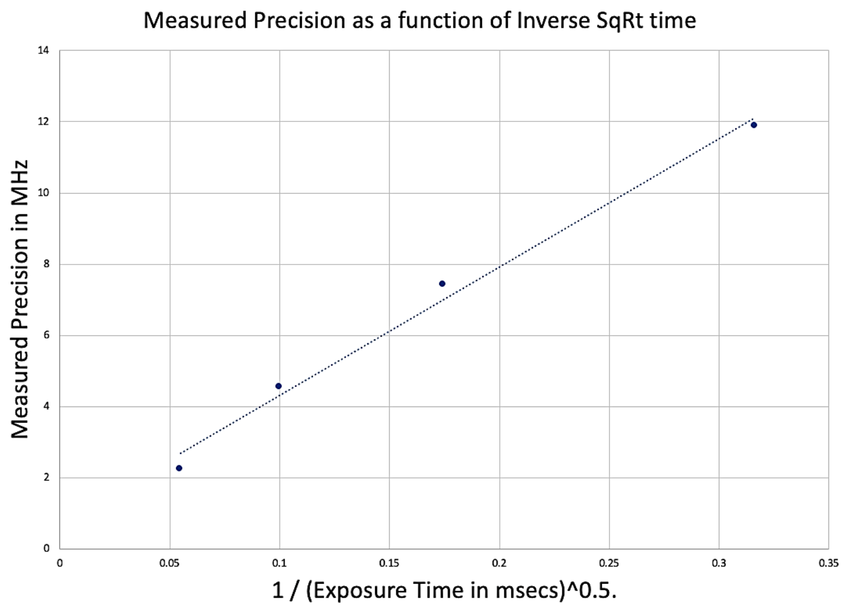

Figure 2. Plot of the measured precision as a function of the inverse of (exposure time)$^{0.5}$. The linear relationship confirms that Equation (B) describes the relationship between precision and Brillouin signal.

Measuring the precision of the Brillouin shift is a somewhat tedious business. If one knows the relationship between the light intensity registered by the sensor pixels and the number of photo-electrons detected by the pixels, then Equation (B) can be used to quickly estimate the precision of any Brillouin shift provided the value of Γ_B (the measured width (FWHM) of the Brillouin peak) is also known [6].

Optimizing the Precision of the measured Brillouin Shift

While the agreement between the theoretical predictions and experimental measurements described above is reassuring, a more practical consideration is how best to utilize a high-resolution spectrometer to optimize the precision of the measured Brillouin shifts. Fortunately, Equation (B) gives a very straightforward answer to this question – *maximizing the value of N will maximize the accuracy of the measured Brillouin shift.* So how does one maximize the total number of Brillouin photo-electrons? A step-by-step process is described below:

1) Maximize the number of pump photons incident upon the sample. As the number of scattered Brillouin photons is proportional to the number of pump photons it is beneficial to both increase the power of the pump laser and to optimize the optical coupling efficiency of the pump photons into the probed volume of the sample.

2) Maximize the number of Brillouin photons collected from the sample. Increasing the efficiency of the collection optics and ensuring that all the collected photons enter the spectrometer will improve the precision of the Brillouin shifts.
3) Ensure that most of the photons entering the spectrometer are incident upon the sensor and that the sensor has a high Quantum Efficiency. (The HF-8999 spectrometer contains a VIPA to ensure high throughput and the sensor has a QE of >80% at the pump wavelength).
4) Use a long exposure time to increase the number of detected photo-electrons. While using a long exposure time will improve the precision of the measured Brillouin shift, it can lead to very long measurement times for scanning systems. Fortunately, the HF-8999 spectrometer records the entire Brillouin spectrum in a single exposure (typically less than one second) allowing high-precision measurements of Brillouin shifts in combination with high-speed X-Y scans. Figure 2 demonstrates the trade-off between exposure times and precision – for example, reducing the exposure time by a factor of 4 will only reduce the precision by a factor of 2.

Conclusions

We show that the precision of a Brillouin shift measurement is proportional to the SNR of the Brillouin signal. Tips are provided for the optimization of this signal, and for estimating the precision of any measurement by simply calculating the total number of photo-electrons contributing to the Brillouin signal.

References and Footnotes

[1] - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6484981/

[2] - https://www.ncbi.nlm.nih.gov/pmc/articles/PMC9073852/

[3] – Looking at Fig 1b of [2], it is clear that shot noise dominates at low background noise whenever the spectral channel size is ~0.2 to 0.5 of the natural Brillouin linewidth. LightMachinery spectrometers usually operate in this regime, with the instrument resolution adjusted to be ~0.2 to 0.5 of the FWHM of the Brillouin linewidth.

[4] – The second term of Equation (A) only becomes important if the tail of the pump signal extends to the frequencies of the Brillouin signals and the shot-noise of this background becomes significant. This is rarely the case with LightMachinery spectrometers due to the use of an ultra-narrow notch filter, the Pump Killer etalon, in the optical system.

[5] – "A Fourier transform spectrometer for the vacuum ultraviolet: design and performance", A P Thorne et al 1987 J. Phys. E: Sci. Instrum. 20 54

[6] – For the measurements shown in Fig. 1 and Fig. 2, we can use the conversion factor supplied by the sensor manufacturer to convert between intensity "counts" and the number of photo-electrons hitting a pixel. Using this conversion factor, we found that the agreement

between the left and right sides of Equation (B) was better than a factor of 2. That is, the precision predicted by dividing the measured FWHM of the Brillouin signal (650 MHz) by $N^{0.5}$ (where N is the total number of photo-electrons in the Brillouin signal) was within a factor of 2 of the measured precision. While the measured agreement was not as exact as one might hope, it does confirm that measuring N is a quick method to estimate the precision of the Brillouin shifts.