TIE-19: Temperature Coefficient of the Refractive Index

0. Introduction

The refractive index of optical glasses changes with temperature, the extend of which depending on the glass type and on the wavelength. It also changes with air pressure, therefore in the following we distinguish between the “refractive index relative to vacuum” or “absolute refractive index” and the refractive index at normal air pressure, called “refractive index relative to normal air” (relative refractive index, all values given in the catalog are relative refractive index values). The standard measurement temperature for refractive index values given on test certificates is 22°C [1]. By taking into account the temperature coefficients of the glasses it is possible to calculate the change of refractive index for any wavelength from near UV to near IR in a temperature range from –100°C up to +140°C. The constants of the formulae are listed in the data sheets of the respective optical glass [2]. The calculation of the actual refractive index at a given temperature often seems not to be straight forward and the technical information no. 29 [3] displays only the formulae for the temperature coefficients for the absolute refractive index. This technical information will give a guideline how to calculate the refractive index at different temperatures. Additionally some results of temperature coefficients for optical glasses will be given and discussed. Some special glasses with negative temperature coefficients in an optical system can help to keep wave front deformations caused by temperature changes on a minimum level. Such glasses are called athermal glasses.

1. Fundamentals

The refractive index of glasses relative to vacuum (absolute refractive index) can be described with sufficient approximation as a function of wavelength from the near IR through the visible to the near UV spectral region by a Sellmeier-Type formula

$$n^2_{abs}(\lambda) - 1 = a \cdot \frac{N}{V} \cdot f \cdot \frac{\lambda^2 \cdot \lambda^2_0}{\lambda^2 - \lambda^2_0} \qquad (1)$$

with:

$n_{abs}(\lambda)$	refractive index relative to vacuum
λ	wavelength of the electromagnetic wave in vacuum
a	constant
N	average number of oscillators in volume V
f	average oscillator strength
λ_0	average resonant wavelength of the electronic oscillator in the UV

The product $a \cdot \frac{N}{V} \cdot f$ as well as λ_0 can be determined for a chosen temperature (for example room temperature) by fitting the measured data for $n_{abs}(\lambda)$ at this temperature. This parameter is temperature independent.

2. Derivation and Characterization of the Formula

The derivative of the Sellmeier function (1) with temperature yields equation (2) after simplification by omitting terms with negligible contributions. The coefficients D_0, D_1, D_2, E_0, E_1 and λ_{TK} for a given glass type will be determined by fitting the experimentally obtained data.

$$\frac{dn_{abs}(\lambda,T)}{dT}=\frac{n^2(\lambda,T_0)-1}{2\cdot n(\lambda,T_0)}\cdot\left(D_0+2\cdot D_1\cdot\Delta T+3\cdot D_2\cdot\Delta T^2+\frac{E_0+2\cdot E_1\cdot\Delta T}{\lambda^2-\lambda_{TK}^2}\right)\qquad(2)$$

with:

T_0	Reference temperature (20°C)
T	Temperature (in °C)
ΔT	Temperature difference versus T_0
λ	Wavelength of the electromagnetic wave in a vacuum (in µm)
D_0, D_1, D_2, E_0, E_1 and λ_{TK}:	constants depending on glass type

The refractive index values given in the optical glass catalog apply for an air pressure of $0.10133\cdot10^6$ Pa. They are called relative refractive index values (n_{rel}). These values can be used for $n(\lambda,T_0)$ in equation (2) with sufficient accuracy.

When the function of temperature is closed integrable, the change in the absolute refractive index, $n_{abs}(l,T_0)$, with the temperature difference ($T-T_0$) can be derived from equation (2):

$$\Delta n_{abs}(\lambda,T)=\frac{n^2(\lambda,T_0)-1}{2\cdot n(\lambda,T_0)}\cdot\left(D_0\cdot\Delta T+D_1\cdot\Delta T^2+D_2\cdot\Delta T^3+\frac{E_0\cdot\Delta T+E_1\cdot\Delta T^2}{\lambda^2-\lambda_{TK}^2}\right)\qquad(3)$$

and

$$n_{abs}(\lambda,T)=n_{abs}(\lambda,T_0)+\Delta n_{abs}(\lambda,T)\qquad(4)$$

Consequently, two simple closed expressions for the calculation of the temperature coefficient of the absolute refractive index: $\frac{dn_{abs}(\lambda,T)}{dT}$, as well as the change of the absolute refractive index with temperature: $\Delta n_{abs}(\lambda,T)$ are available.

The refractive index relative to air $\frac{dn_{rel}(\lambda,T)}{dT}$ and $\Delta n_{rel}(\lambda,T)$ can be calculated from:

$$n_{rel}(\lambda,T)=\frac{n_{abs}(\lambda,T)}{n_{air}(\lambda,T,p)}\qquad(5)$$

and

$$\frac{dn_{abs}(\lambda,T)}{dT} = n_{air}(\lambda,T,p)\cdot\frac{dn_{rel}(\lambda,T)}{dT} + n_{rel}(\lambda,T)\cdot\frac{dn_{air}(\lambda,T,p)}{dT} \qquad (6)$$

or

$$\frac{dn_{rel}(\lambda,T)}{dT} = \frac{\frac{dn_{abs}(\lambda,T)}{dT} - n_{rel}(\lambda,T)\cdot\frac{dn_{air}(\lambda,T,p)}{dT}}{n_{air}(\lambda,T,p)} \qquad (7)$$

In (6) and (7) $n_{rel}(\lambda,T_0)$ can be substituted for $n_{rel}(\lambda,T)$ with sufficient accuracy.

The value for $n_{air}(\lambda,T,p)$ and $\frac{dn_{air}(\lambda,T,p)}{dT}$ can be calculated with good accuracy with

$$n_{air}(\lambda,T,P) = 1 + \frac{n_{air}(\lambda,15°C,P_0) - 1}{\left(1 + 3.4785\times10^{-3}\frac{1}{°C}\cdot(T - 15°C)\right)}\cdot\frac{P}{P_0} \qquad (8)$$

$$n_{air}(\lambda,15°C,P_0) = 1 + \left(6432.8 + \frac{2949810\frac{1}{\mu m^2}\cdot\lambda^2}{\left(146\frac{1}{\mu m^2}\cdot\lambda^2 - 1\right)} + \frac{25540\frac{1}{\mu m^2}\cdot\lambda^2}{\left(41\frac{1}{\mu m^2}\cdot\lambda^2 - 1\right)}\right)\times10^{-8} \qquad (9)$$

$$\frac{dn_{air}(\lambda,T,P)}{dT} = -0.00367\cdot\frac{n_{air}(\lambda,T,P) - 1}{1 + 0.00367\frac{1}{°C}\cdot T} \qquad (10)$$

with:

P_0	$0.101325\mathrm{x}10^6$ Pa (normal pressure in Pascal)
P	air pressure
λ	Wavelength of the electromagnetic wave in a vacuum (in µm)
T	temperature (in °C)

With the help of 6 glass specific parameters D0, D1, D2, E0, E1 and λ_{TK} the value of the temperature coefficient of the absolute refractive index (compared to vacuum) as a function of temperature and wavelength can be calculated with equation (2) for each glass type. The variation of the absolute refractive index compared to the value at 20°C can be calculated with equation (3). The corresponding value for air can be calculated by (7) and (5).

The constants of formula (2) and (3) are given in the glass data sheets [2]. The constants are valid for a temperature range from –100°C to +140°C and a wavelength range from 0.3650 µm to 1.014 µm. The temperature coefficients in the data sheets are guideline values.

Upon request, measurements can be performed on individual melts for the temperature and wavelength range as given above with a precision better than $\pm 5 \cdot 10^{-7}$/K. The constants of the dispersion formula can be calculated from this measured data and will be listed on the test certificate for the individual glass melt on request.

3. Temperature Coefficients of Optical Glasses

The extent of change of the refractive index at a given wavelength varies from glass to glass. Table 1 shows the list of our current product range sorted by the relative temperature coefficients between +20°C and +40°C at the e-line (~546.1 nm). The values reach from $+12.5*10^{-6}$/K for SF57 down to $–6.7*10^{-6}$/K for N-PK51 and $–10.4*10^{-6}$/K for LITHOTEC-CAF2.

Glass			**Relative temperature coefficients of refractive index [+20/+40 deg.C] in 10^{-6}/K**		
	nd	**Vd**	**1060nm**	**e-line**	**g-line**
SF57	1,84666	23,83	7,60	12,50	18,90
SF57HHT	1,84666	23,83	7,60	12,50	18,90
SF6	1,80518	25,43	6,80	11,10	16,20
SF6HT	1,80518	25,43	6,80	11,10	16,20
SF56A	1,78470	26,08	6,20	10,00	14,70
LITHOSIL-Q	1,45844	67,83	9,40	9,90	10,40
N-LASF40	1,83404	37,30	7,30	9,30	11,40
SF4	1,75520	27,58	5,70	9,20	13,30
N-LAF36	1,79952	42,37	7,40	9,10	10,80
P-LASF47	1,80610	40,90	6,90	8,60	10,30
N-LAF33	1,78582	44,05	7,00	8,50	10,00
N-LAF35	1,74330	49,40	7,10	8,40	9,60
LAFN7	1,74950	34,95	6,30	8,30	10,40
SF10	1,72825	28,41	5,30	8,10	11,60
SF1	1,71736	29,51	5,00	7,90	11,30
N-ZK7	1,50847	61,19	6,40	7,00	7,60
N-LASF43	1,80610	40,61	5,00	6,50	8,10
P-SF67	1,90680	21,40	2,80	6,30	11,70
SF5	1,67270	32,21	3,50	5,80	8,40
KZFS12	1,69600	36,29	4,30	5,70	7,30
N-LASF45	1,80107	34,97	3,80	5,70	7,90
N-LASF41	1,83501	43,13	4,00	5,40	6,80
N-LAF34	1,77250	49,62	4,30	5,40	6,50
N-KZFS2	1,55836	54,01	4,70	5,30	5,90
N-LASF44	1,80420	46,50	4,00	5,30	6,50
N-SSK2	1,62229	53,27	4,30	5,20	6,10
N-BALF4	1,57956	53,87	4,20	5,10	6,00

N-LAK10	1,72003	50,62	4,20	5,10	6,10
N-LASF9	1,85025	32,17	2,90	5,10	7,70
N-LAF21	1,78800	47,49	3,90	5,10	6,20
N-LAK8	1,71300	53,83	4,10	5,00	5,80
N-BAF10	1,67003	47,11	3,80	4,90	6,00
N-LASF31A	1,88300	40,76	3,30	4,90	6,60
SF2	1,64769	33,85	2,70	4,60	6,90
F5	1,60342	38,03	3,00	4,60	6,20
N-KZFS11	1,63775	42,41	3,50	4,60	5,70
N-SK2	1,60738	56,65	3,60	4,50	5,30
F2	1,62004	36,37	2,70	4,40	6,30
N-BASF2	1,66446	36,00	2,90	4,40	6,20
N-LAK33A	1,75393	52,27	3,40	4,40	5,30
N-BASF64	1,70400	39,38	2,80	4,30	5,90
N-LAF7	1,74950	34,82	2,60	4,30	6,30
K10	1,50137	56,41	3,60	4,20	4,90
N-LAK14	1,69680	55,41	3,20	4,00	4,70
N-KZFS8	1,72047	34,70	2,40	4,00	5,80
N-BAK4	1,56883	55,98	3,10	3,90	4,70
N-LAK34	1,72916	54,50	3,00	3,80	4,60
N-SK5	1,58913	61,27	3,20	3,70	4,30
N-KZFS4	1,61336	44,49	2,70	3,70	4,70
N-LAK9	1,69100	54,71	2,90	3,70	4,40
P-SK57	1,58700	59,60	2,90	3,60	4,30
N-F2	1,62005	36,43	2,10	3,50	5,10
N-BK10	1,49782	66,95	2,90	3,40	3,80
N-SF15	1,69892	30,20	1,60	3,40	5,80
N-SF5	1,67271	32,25	1,80	3,40	5,50
N-BAF4	1,60568	43,72	2,20	3,30	4,50
N-BAF52	1,60863	46,60	2,30	3,30	4,30
N-SSK5	1,65844	50,88	2,20	3,20	4,20
N-SK11	1,56384	60,80	2,60	3,20	3,80
N-LAK22	1,65113	55,89	2,40	3,10	3,90
N-SK14	1,60311	60,60	2,40	3,10	3,70
N-PSK3	1,55232	63,46	2,50	3,00	3,50
N-BK7	1,51680	64,17	2,40	3,00	3,50
N-BALF5	1,54739	53,63	2,10	2,90	3,70
N-SSK8	1,61773	49,83	2,00	2,90	3,90
LLF1	1,54814	45,75	1,90	2,90	3,90
N-BAF51	1,65224	44,96	1,70	2,90	4,10
N-SK4	1,61272	58,63	2,10	2,80	3,40
N-SF64	1,70591	30,23	0,90	2,70	5,10
N-SF8	1,68894	31,31	0,90	2,60	4,80
N-BAK1	1,57250	57,55	1,80	2,50	3,20
N-SF66	1,92286	20,88	-0,50	2,40	7,30
N-SF11	1,78472	25,68	0,10	2,40	5,60
N-SK16	1,62041	60,32	1,70	2,30	2,90

N-SF57	1,84666	23,78	-0,50	2,20	6,00
N-SF57HT	1,84666	23,78	-0,50	2,20	6,00
N-K5	1,52249	59,48	1,40	2,10	2,70
N-SF56	1,78470	26,10	-0,30	2,00	5,10
LF5	1,58144	40,85	0,80	2,00	3,40
N-SF1	1,71736	29,62	0,00	1,80	4,20
N-KF9	1,52346	51,54	0,90	1,80	2,60
N-BAK2	1,53996	59,71	1,00	1,70	2,30
K7	1,51112	60,41	0,90	1,60	2,20
N-SF6	1,80518	25,36	-0,80	1,50	4,80
N-SF6HT	1,80518	25,36	-0,80	1,50	4,80
N-SF10	1,72828	28,53	-0,50	1,50	4,10
N-SF4	1,75513	27,38	-0,70	1,40	4,20
N-SF14	1,76182	26,53	-1,10	1,10	4,10
N-LAK21	1,64049	60,10	0,50	1,00	1,60
N-LAF2	1,74397	44,85	-0,10	1,00	2,30
N-LAK7	1,65160	58,52	0,00	0,70	1,30
N-LAK12	1,67790	55,20	-1,20	-0,40	0,30
N-FK5	1,48749	70,41	-1,40	-1,00	-0,60
N-PSK53	1,62014	63,48	-2,90	-2,30	-1,80
N-PSK53A	1,61800	63,39	-2,90	-2,40	-1,80
P-PK53	1,52690	66,22	-5,60	-5,20	-4,70
N-FK51A	1,48656	84,47	-6,00	-5,70	-5,30
N-PK52A	1,49700	81,61	-6,70	-6,40	-6,00
N-PK51	1,52855	76,98	-7,10	-6,70	-6,40
LITHOTEC-CAF2	1,43385	95,23	-10,40	-10,20	-9,90

Table 1: List of our current product range sorted by the relative temperature coefficients between +20°C and +40°C at the e-line (~546.1 nm)

Figures 1 to 4 show the change of the temperature coefficients of refractive index (absolute on the left side and relative on the right side) with temperature for different wavelengths and different glasses. It can be seen that the shape of the curve of the relative temperature coefficients differs from the absolute temperature coefficients. The reason is the influence of the temperature coefficients of the air. In general the influence of the temperature on the refractive index at shorter wavelengths is much higher compared to longer wavelengths. This "dispersion" of the temperature coefficient is more pronounced for high dispersion glasses.

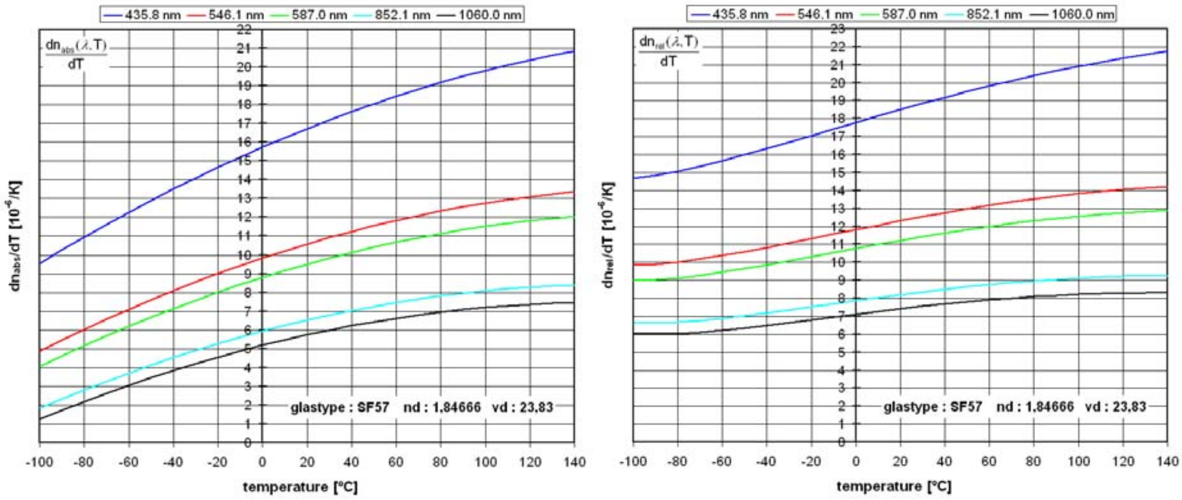

Figure 1: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of SF57 for different wavelengths

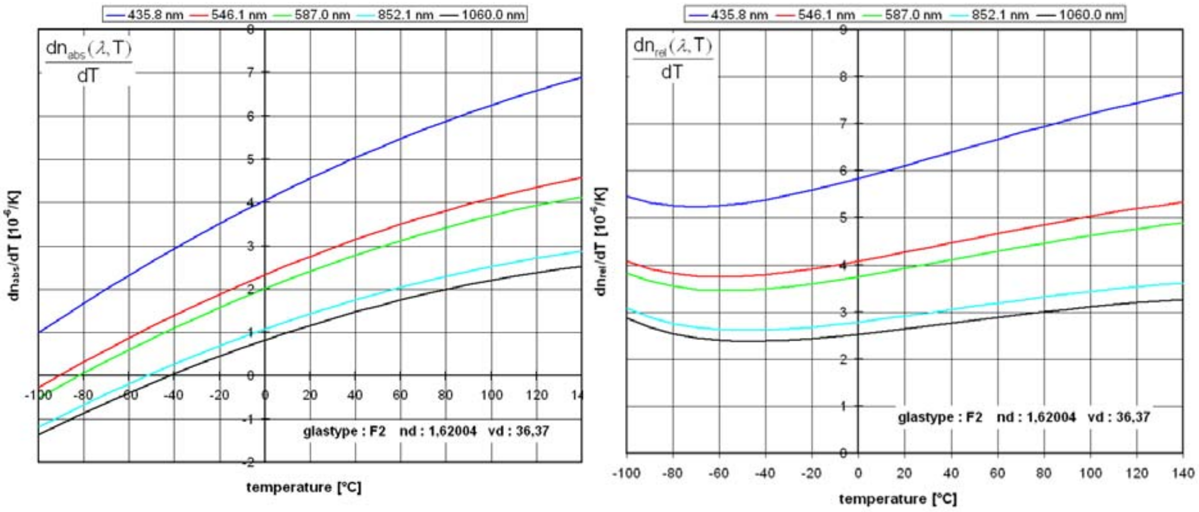

Figure 2: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of F2 for different wavelengths

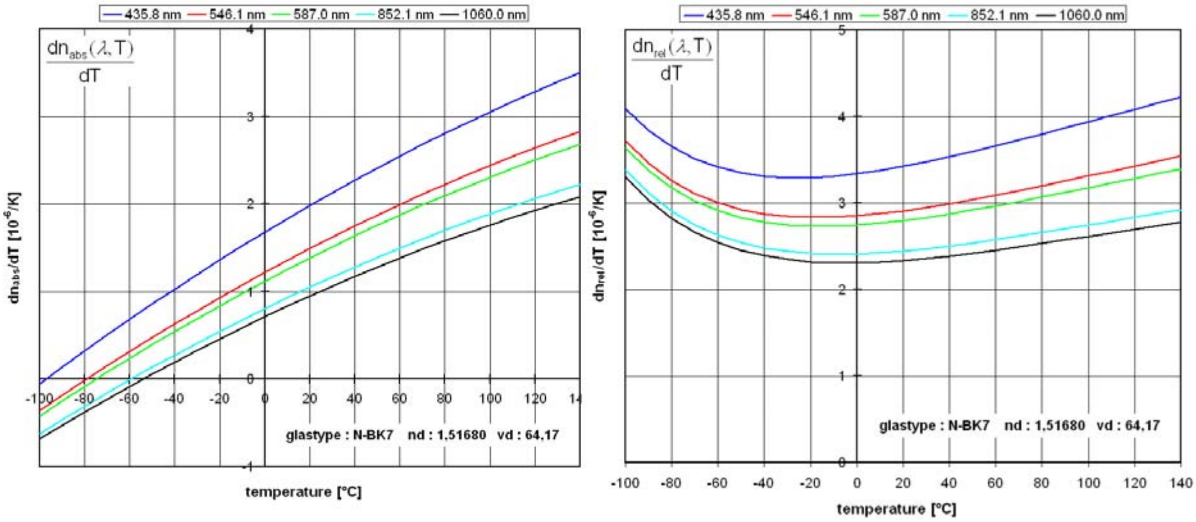

Figure 3: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **N-BK7** for different wavelengths

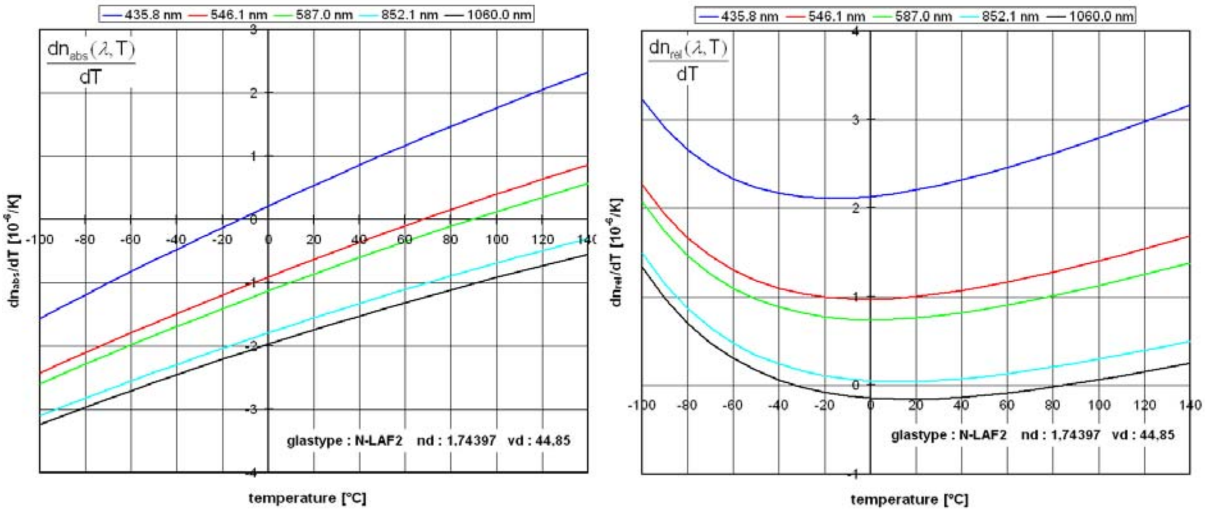

Figure 4: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of **N-LAF2** for different wavelengths

Figure 5 summarizes the above displayed examples in one diagram to better pronounce the different temperature coefficients of different glasses. Often it is easier to display the absolute values of refractive index as a function of temperature. In figure 7 the change of the relative refractive index with temperature is displayed for the above discussed glass types. It can be seen that a temperature increase of 20°C changes the refractive index of SF57 at the d-line by $2{,}3 \cdot 10^{-4}$. Additionally the diagrams also contain the temperature change curves of N-PK51. This glass has a negative temperature coefficient over the complete temperature range.

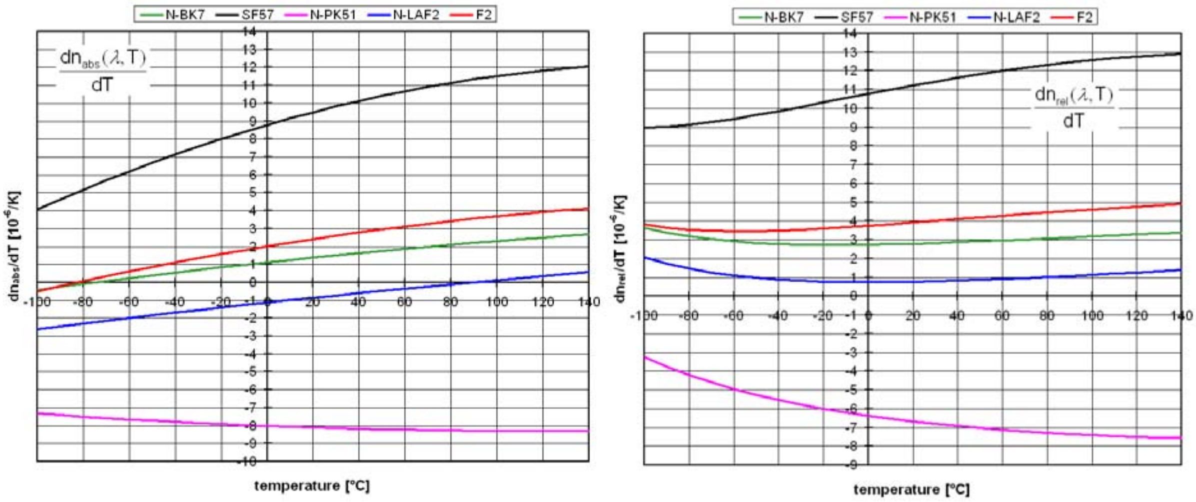

Figure 5: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of different glass types in comparison.

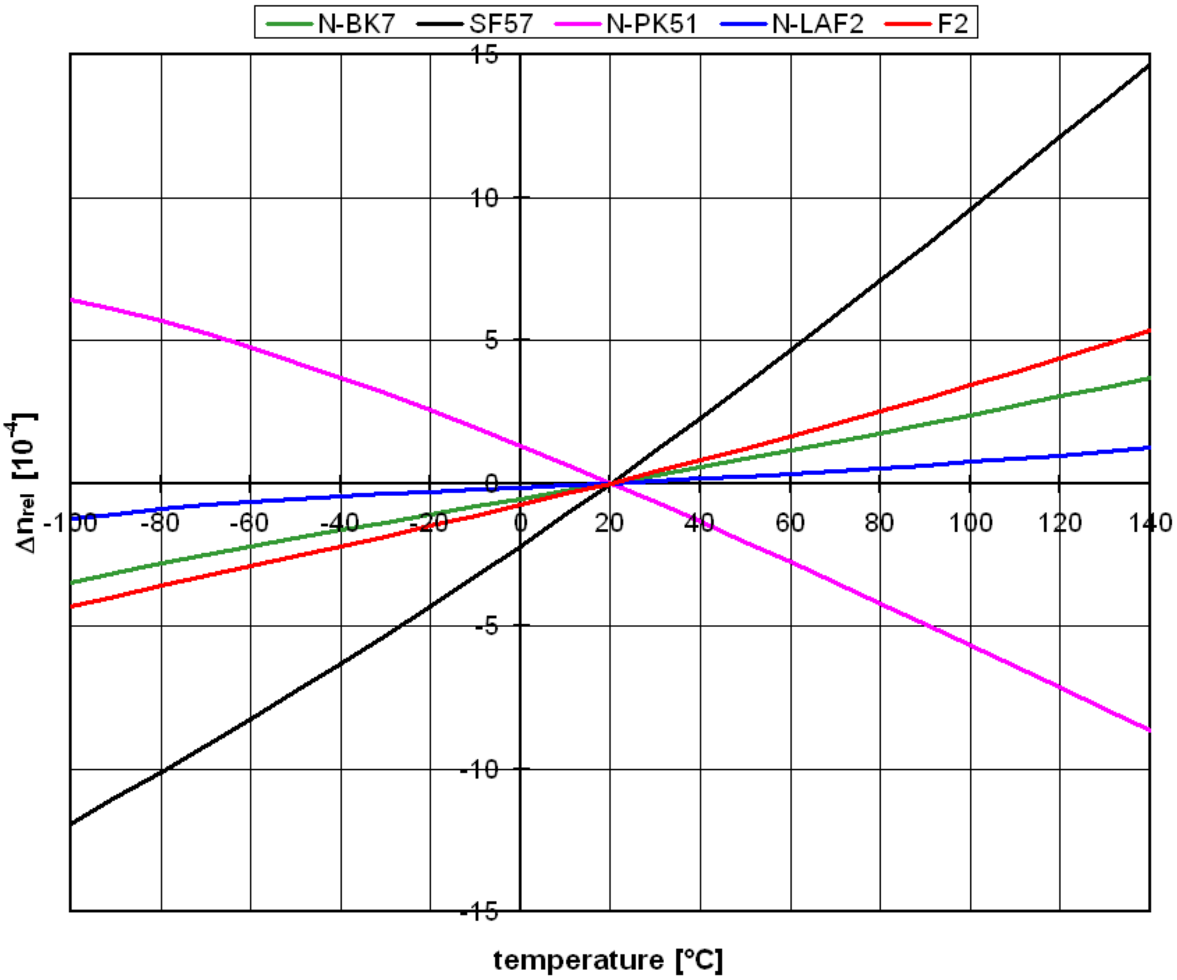

Figure 6: Change of the relative refractive index (catalog value at 20°C) with temperature of some different glass types.

4. Athermal Glasses

In conditions, where the optical system is subjected to varying inhomogeneous temperature environments, wave front distortions can be expected due to changes in optical path length and refractive index with temperature.

To compensate such effects, it is important that the effect of wave front distortions generated due to the inhomogeneous thermal expansion of the glass can be compensated by the refractive index change with temperature. The fact that the thermal expansion coefficient of optical glasses is always positive implies the need of optical glasses with a negative temperature coefficient to compensate thermal effect on the wave front. This can be expressed in the following equation:

$$G = \alpha \cdot (n_{rel}(\lambda, T) - 1) + \frac{dn_{rel}(\lambda, T)}{dT} \qquad (11)$$

α is the thermal expansion coefficient of the glass, $n_{rel}(\lambda,T)$ the refractive index at the proposed wavelength. The value thermo optical constant G should be as low as possible. Glasses with an value of G near zero are called athermal glasses. Table 2 shows that not all glasses exhibiting a negative temperature coefficient are excellent athermal glasses. Optical glasses with negative temperature coefficients are N-FK5, N-PSK53, N-FK51A, N-PK52A and the precision molding glass P-PK53 and also LITHOTEC-CaF2. But only N-PK51, N-FK51A and N-PK52A show a real athermal behavior.

Glass	**n_e**	**Relative temperature coefficients of refractive index [+20/+40 deg.C], e-Line [$10^{-6}K^{-1}$]**	**alpha -30/70 [$10^{-6}K^{-1}$]**	**G [$10^{-6}K^{-1}$]**
LITHOTEC-CAF2	1,43494	-10,20	18,41	-2,19
N-PK51	**1,53019**	**-6,70**	**12,35**	**-0,15**
N-PK52A	**1,49845**	**-6,40**	**13,01**	**0,08**
N-FK51A	**1,48794**	**-5,70**	**12,74**	**0,52**
P-PK53	**1,52880**	**-5,20**	**13,31**	**1,84**
N-FK5	1,48914	-1,00	9,2	3,50
N-PSK53A	1,62033	-2,40	9,56	3,53
N-PSK53	1,62247	-2,30	9,4	3,55
N-LAK12	1,68083	-0,40	7,6	4,77
N-BK7	1,51872	3,00	7,1	6,68
N-LAF2	1,74791	1,00	8,06	7,03
F2	1,62408	4,40	8,2	9,52
SF57	1,85504	12,50	8,3	19,60

Table 2: Optical glasses and their thermo optical constant G.

Figure 7 shows the temperature coefficient of N-PK51. It can be seen that the values are negative over the complete temperature region from –100°C to +140°C.

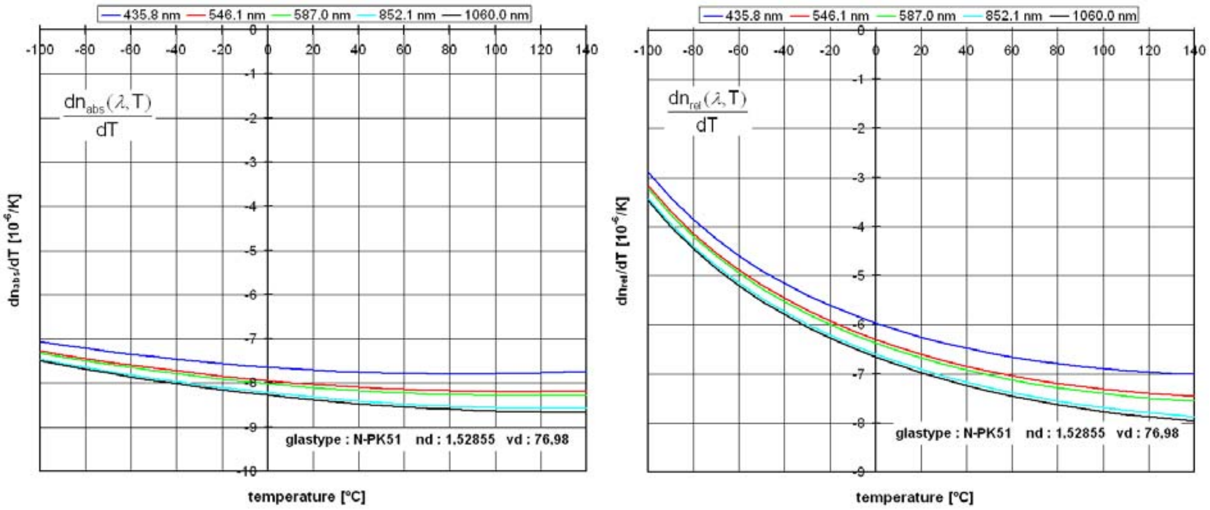

Figure 7: Temperature coefficient of the absolute (left figure) and relative (right figure) refractive index of N-PK51 for different wavelengths

P-PK53 shows the lowest athermal constant of all precision molding glasses. The thermo optical constant of N-BK7 and SF57 is much higher.

5. Literature

[1] SCHOTT Optical Glass Pocket Catalog 2007
[2]www.us.schott.com/advanced_optics/english/our_products/materials/data_tools/index.html
[3] SCHOTT Technical Information TIE-29 “Refractive index and dispersion”

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